Indian Statistical Institute
Bangalore Centre
B.Math (Hons.) III Year 2010-2011
Second Semester
Sample Survey and Design of Experiments

Mid-semester Examination   Date : 4.3.11

Answer as many questions as possible. The maximum you can score is 60.
All the notation have their usual meaning. State clearly the results you
use.

1. Consider a simple random sample (SRS) of size n from a population
of size N, collected without replacement (WOR).

(a) Show that the sample mean is unbiased for population mean.

(b) Obtain the expression for variance of the sample mean.

(c) In order to estimate the proportion of smokers in a region with
25723 adult males, a simple random sample of size 200 was taken
without replacement. 87 of them were found to be smokers.

(i) Estimate the proportion of smokers in the population and find the
standard error of your estimate.

(ii) Do the results of the sample furnish conclusive evidence that the
majority of the adult males are non-smokers.

[State clearly your assumption(s) with a short justification.]

$$[3 + 5 + ((2+3) + 4 = 17]$$

2. (a) A sampler decides to take a stratified random sample with total
sample size n. The cost function is known to be of the form $C = c_0 + \sum t_h \sqrt{n_h}$ where c_0 and the t_h are known numbers.

Find the number of units (n_h) to be chosen from the hth stratum,
$h = 1, 2, \cdots k$, such that that $V(\bar{y}_{st})$ is minimum, assuming that the
total cost is fixed.

(b) A sampler decided to take a stratified random sample. His advance
estimates of relevant quantities for the two strata were as follows.

Stratum	N_h	S_h	C_h (in rupees)
1	413	10	4
2	617	20	9

(i) Find the values of n_1/n and n_2/n that would minimize $V(\bar{y}_{st})$.

(ii) How much would the total field cost be?

(iii) After the sample was taken, the sampler found that his field cost
per unit were actually Rs 2 in stratum 1 and Rs 12 in stratum 2. How
much greater was the field cost than anticipated?

$$[\,7 + (3+2+3) = 15\,]$$

3. A factory owner wants to see whether 4 heating elements produce the
same amount of heat . He decided to take 2 observations each for

the first two elements as they were old. Further, financial restrictions allowed him to take 10 observations altogether.

(a) Describe how the experiment is to be conducted.

(b) Formulate this as a testing of hypothesis problem and derive a suitable test procedure.

(c) If it is found that all heating elements are not performing in the same way, it is required to find exactly which elements differ. Suggest a computational procedure to do that with justification. $[3 + 6 + 4 = 13]$

4. A manufacturer suspects that the raw material supplied in different batches differ in calcium content.

(a) He asked three chemists to analyse the materials. Four batches of raw material were taken and a randomised block design(RBD) was used.

(i)Write down an appropriate model.

(ii) Derive the least square estimator of the difference in the calcium contents of different batches.

(iii) Derive the test statistics for testing whether the amount of calcium differ significantly in different batches.

(b) Suppose due to financial restrictions, an RBD could not be used. Instead the experiment was conducted in the following manner.

Chemist 1 handled 2 samples from batch 1, 1 from batch 2.

Chemist 2 handled 1 sample from each of batch 2 and batch 3.

Chemist 3 handled 1 sample from batch 3 and two from batch 4.

Suppose the manufacturer wants to estimate the difference in the calcium contents of batch 1 and batch 4. Is the estimator derived in Q (a)(ii) is a valid one ? Give reasons. If not provide (i) an unbiased estimator and (ii) the BLUE.

$$[(2 + 3 + 4) + (1 + 3 + (2+4)) = 19]$$